MID-SEMESTER EXAMINATION
COMPLEX ANALYSIS, B. MATH III YEAR
I SEMESTER, 2012-2013

The eight questions below carry a total of 110 marks. Answer as many questions as you can. The maximum you can score is 100 and the time limit is 3 hours.

Notations: Ω stands for an open connected subset of $\mathbb{C}, U$ for $\{z \in \mathbb{C} : |z| < 1\}$ and T for the boundary of U.

1. If f is an entire function such that $|f(z)| \leq 1 + |z|^2$ whenever $|z| > 1$ prove that $\frac{d^3}{dz^3} f(1) = 0$. [15]

2. Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ where the series converges for $|z| < 1$. If $f(\frac{1}{n}) \in \mathbb{R}$ for $n = 2, 3, , ...$ show that $f(\mathbb{R}) \subset \mathbb{R}$. [10]

3. Let d be the usual metric on $\mathbb{C}_\infty$ defined in terms of the stereographic projection. Compute $d(i, \infty)$. [15]

4. Without using any theorem of Complex Analysis show that polynomials have mean value property. [15]

5. If $f, g, \bar{f}g \in H(\Omega)$ show that $g \equiv 0$ or else f is a constant. [15]

6. Prove the following statement using Open mapping Theorem but not Maximum Modulus Principle:

If $f \in H(\Omega), c \in \Omega$ and $|f(c)| \geq |f(z)|$ $\forall z \in \Omega$ then f is necessarily a constant. [15]

7. Prove that the infinite product $\prod_{n=1}^{\infty} (1 + \frac{z}{n^2} - \frac{z^2}{n^3})$ conveges uniformly on compact subsets of $\mathbb{C}$ to an entire function and find the zeros of f. [10]

8. Let Let $f(z) = \frac{2z-i}{2+iz}$. What is the largest open set on which f is holomorphic? Show that f maps U onto U and T onto T. [15]